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14. LEIBNIZ AND RUSSELL

*The number of all numbers and the set of all sets**

1. INTRODUCTION

In *My Philosophical Development* Bertrand Russell wrote:

> There is one major division in my philosophical work: in the years 1899–1900 I adapted the philosophy of logical atomism and the technique of Peano in mathematical logic. This was so great a revolution as to make my previous work, except such as was purely mathematical, irrelevant to anything that I did later. The change in these years was a revolution; subsequent changes have been of a nature of evolution. (Russell 1959: 11)

It is well known that, precisely during these years, Russell was intensively occupied in lecturing and writing a book on Leibniz. It is less well known to what extent and in what ways Leibniz's work influenced Russell. It is fairly clear, nonetheless, that his engagement with Leibniz's philosophy significantly influenced Russell – both in positive and negative ways. For example, Russell noted that it was through his study of Leibniz that he realized the importance of relations. The logic of relations is one of Russell's most significant contributions to logic and philosophy in general, and, it goes without saying, to that of the English-speaking world in particular. Incidentally, "The Logic of Relations" was Russell's early title for what was later titled *Principia Mathematica* (1903).

It is also largely due to Russell's pioneering role that philosophy in the English-speaking world has become mathematical or, as we have come call it, analytical.[1] Although Leibniz might have been a source of inspiration for the establishment of a mathematically oriented philosophy,[2] Leibniz was the "other" with respect to Russell's logic of relations. With regard to Russell's (and Frege's) fundamental innovative logic, namely that all predicates are types of relations, some being monadic and some being polyadic, Leibniz was the adversary.[3] At the time that Russell was producing his book on Leibniz he set himself two major objectives: (1) to reduce mathematics to logic and (2) to replace the traditional subject-predicate logic with his own logic of relations (that is, to analyze predication by using propositional functions). There is little doubt that it was in light of these objectives that Russell wrote:

> I found what so many books on Leibniz failed to make clear – that his metaphysic was explicitly based upon the doctrine that every proposition attributes a predicate to a subject and (what

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seemed to him almost the same thing) that every fact consists of a substance having a property. I found that this same doctrine underlies the systems of Spinoza, Hegel, and Bradley […]. (Russell 1959: 61)

A substantial portion of Leibniz's scholarship in the twentieth century was devoted to grappling with this claim – showing that, however insightful and interesting it may be, it is strongly overstated, if not entirely misguided. What led Russell to this view of Leibniz's system wasn't just his attempt to reduce all notions to logical ones (his logicism) but also the role Leibniz played in his philosophical agenda of refuting the traditional subject-predicate Logic. Russell argued that Leibniz's commitment to the subject-predicate logic, on the one hand, and his use of relations, on the other, rendered his system incoherent.[4] In this way, Leibniz's work provided Russell with a formulation of the doctrine he was trying to refute and to produce an alternative for. Although Russell's agenda led him to serious misinterpretations of Leibniz (which are familiar by now), Russell and Frege's Logic of relations has known a great success, so much so that it has become the common language of contemporary philosophers and logicians.[5]

As far as the logic of relations is concerned, Leibniz played the role of the "other" – an illustration why an altogether different approach to relations is needed. In this respect, Russell's agenda was a very successful one. Curiously enough, it seems that Leibniz played quite a different role in the development of Russell's agenda and, consequently, in the development of logic, mathematics and philosophy in the English-speaking world.

2. THE NUMBER OF ALL NUMBERS AND THE CLASS OF ALL CLASSES

It is well known that Russell's logicist programme (presented in *Principles of Mathematics*, 1903) to reduce all mathematical notions to logical ones was unsettled by a paradox. Frege showed how to define numbers by means of classes. Russell noticed that this gives rise to the notion of the class of all classes. He then asked whether this class is a member in the class of all classes (or in it self). As it turns out, both options, viz., that it is a member of itself or not, are impossible. Hence, the definition of numbers in terms of classes seems to generate a paradox. Russell himself did not at first see the devastating effect of the paradox he had discovered. This was conveyed to him in a letter from Frege who wrote: "not only the foundations of my arithmetic, but the sole possible foundations of arithmetic, seem to vanish."[6] As Lavine remarks, this is how Russell's "conundrums" became Russell's Paradox and changed the course of his agenda.

Whether the paradox constitutes a problem for Cantor's set theory or for set theory in general is a controversial matter (see especially Lavine 1994). What is clear beyond any controversy is that the paradox had enormous influence on Russell's philosophy, on the development of set theory, on logic and on the project of providing a foundation for mathematics and for knowledge through logic.

I will therefore assume that the impact of Russell's paradox on Anglo American philosophy requires no further argument.

Let us first take note of some reference points in the chronology of Russell's discovery of the paradox:[7]

- In 1895 Cantor discovered that the assumption that the system of all numbers is a set leads to contradictions. He called such a set an inconsistent absolutely infinite multiplicity.[8]
- In 1896, Russell learned about Cantor's work.[9]
- During 1899–1900 Russell was writing his book on Leibniz and translating passages from the Latin into English for the appendix to his book.[10]
- By early March 1900, Russell completed his *Critical Exposition of the Philosophy of Leibniz* (RCP liii).
- In May 1901, Russell discovered a paradox but, as Lavine remarks, "he did not discuss at the time the class of all classes that are not members of themselves, but only the class of all predicates that cannot be predicated of themselves. The class version ('There is no class of all classes that do not belong to themselves as members') appears only a year later in his letter to Frege" (Lavine 1994: 60–1 note 21).
- In June 1902, Russell wrote to Frege and Peano about the paradox.
- While Frege was devastated by Russell's Paradox, Russell set out to develop his theory of Types in an attempt to avoid the paradox.

Long before he first formulated "Russell's paradox" in May 1901, in 1899 and 1900, Russell was preoccupied with what he variously called "the contradiction of infinity" or "the antinomy of the greatest number." For example, in a draft of the *Principles of Mathematics* (of 1899–1900) he wrote:

> Mathematical ideas are almost all infected with one great contradiction. This is the contradiction of infinity. All antinomies, I believe, so far as they are valid at all, will be found reducible to the antinomy of infinite number. (RCP vol. III, Paper #1, 70 and 11)

As G. H. Moore notes in his introduction to *The Collected Papers of Bertrand Russell*, "In 1899, in 'Fundamental Ideas', he [Russell] had been very concerned with the antinomy of infinite number. He wrote that the totality for 'classes seems necessary; but if we make it so, infinite number with its contradictions becomes inevitable, being the number of concepts or of numbers" (Russell 1899b: 266).[11] In the same chapter Russell spells out the antinomy as follows: "there are many numbers, therefore there is a number of numbers. If this be N, N + 1 is also a number, therefore there is no number of numbers" (Russell 1899b 265 and xxi).[12] This reasoning gives rise to Russell's summary of the Antinomy of Infinite Number (in Fundamental Ideas, Chapter VII), as follows: "This [the Antinomy] arises most simply from applying the idea of a totality to numbers. There is, and is not, a number of numbers" (Russell 1899b: 267/RCP xx).

AQ1

AQ2

The formulations in terms of the number of all numbers will ring familiar to Leibniz's readers (and especially to readers of his early texts). Leibniz explicitly and quite frequently stated that "The number of all numbers is a contradiction"

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(e.g. A VI iii, 463/P 7). We already noted that Russell was studying and translating Leibniz's texts intensively at the time. In his book on Leibniz, Russell cites him as saying that the number of all numbers implies a contradiction[13] and adds: "Leibniz denied infinite *number*, and supported his denial with very sound arguments."[14]

As Russell notes, Leibniz not only stated that the number of all numbers is a contradiction but also explained it in some detail. Although his arguments are not perspicuous, they are worth quoting. For example, in his Paris notes, he wrote:

> The number of finite numbers cannot be infinite; from which it follows that there cannot be an infinity of square numbers, taken in order starting from one. From which it seems to follow that an infinite number is impossible. It seems that one only has to prove that the number of finite numbers cannot be infinite. If the numbers are assumed to exceed each other continuously by one, the number of such finite numbers cannot be infinite, for in that case the number of numbers is equal to the greatest number, which is assumed to be finite. It has to be replied that there is no greatest number. But even if they were to increase in some way other than by ones, yet if they always increase by finite differences, it is necessary that the number of all numbers always has a finite ratio to the last number; further, the last number will always be greater than the number of all numbers. From which it follows that the number of all numbers is not infinite; neither, therefore, is the number of units. Therefore there is no infinite number, or, such a number is not possible. (A VI iii, 477/ P 31–3)[15]

In alluding to Galileo's paradox (that there are as many natural numbers as square numbers), Leibniz is explicitly arguing here that the number of all finite numbers cannot be a number since it cannot be either finite or infinite. Therefore, he concludes that such a notion is contradictory or impossible.

Given the obvious similarity between the number of all numbers and the class of all classes, it is interesting to consider the relation between Leibniz's formulations of the impossibility of the number of all numbers and Russell's paradox. It is significant in this regard that Russell has three different formulations of the paradox. As we have seen, he first formulated the paradox (or antinomy) in terms of numbers. He then formulated it in terms of predicates, namely, "the predicate of a predicate being impredicable of itself" (in RCP xxxvii); and finally in the now familiar terms of classes (the class of all classes that do not belong to themselves). The first formulation in terms of numbers sounds very Leibnizian. The second, in terms of predicates, also has interesting relations to Leibniz.

If one recalls Leibniz's use of the *in-esse* principle, that is, the inclusion of a predicate in a subject, a principle which Russell placed at the center of Leibniz's philosophy, and if one interprets the notion of a number of all numbers in these terms, i.e., a number, seen as a subject term including all numbers as its proper predicates, it becomes clear that such a number has to include itself as a predicate. If this number be N, then it is clear that it does not include $N + 1$ as a predicate. But, if so, it is not the number of all numbers for there is a number that it does not include.

The third formulation in terms of classes, with the precise definitions of classes and of the membership-relation is altogether foreign to the seventeenth century context. These notions were spelled out rigorously by Cantor. The concepts introduced in Cantor's set theory clearly stand between Leibniz notion of the greatest number and Russell's Paradox. Furthermore, this formulation utilizes classes in an extensional sense (which was foreign to Leibniz). It seems reasonable to suppose that Russell's paraphrase of the paradox in extensional terms (i.e., in terms of classes) is related to his attempt to engage Frege in the problem (as testified by his letter to him). Russell's initial formulation in terms of predicates does not make sense in the context of Frege's system. In order to engage Frege, he had to formulate the problem in extensional terms.[16]

Yet it seems that the intuition, as well as the basic logical structure of the paradox, can be read into Leibniz's formulations, that is, *if* they are interpreted with Cantorian concepts in mind. It is therefore arguable that Russell's paradox could be generated by applying Cantor's set theory to Leibniz's notion of the number of all numbers. This is the exercise I propose to engage in.

Let me be very clear on this point. I do not suggest that Russell's paradox was invented by Leibniz. Such a claim is not only false but is a significant distortion in the sense that it ignores the subtle historical context in which both Leibniz and Russell's formulations arise. Yet the syntactical similarity between the number of all numbers and the class of all classes is striking and suggestive. It is an interesting exercise, therefore, to bring out points of similarity as well as points of difference between Leibniz's claim that the number of all numbers is a contradiction and Russell's paradox.

A particularly interesting question in this regard is whether the self-reflexivity of Russell's formulation, (viz. the class of all classes which is not a member of itself) is already implicit in Leibniz's formulations. From a current perspective, it seems that Leibniz's observation that the notion of the number of all numbers is inconsistent anticipates some of the dramatic consequences and discoveries about number, infinity, and some well-known paradoxes. For example, Leibniz's argument that "there is no infinite number" or that "the number of all numbers is not a number" anticipates Cantor's treatment of infinite cardinality as powers rather than as numbers in the traditional sense. More generally, Leibniz's argument points to the insight that infinite magnitudes cannot be quantified by numbers or cannot be expressed by numbers. One can see Cantor's development of infinite cardinality as closely related to Leibniz's arguments that the number of all numbers is not a number.

One can also see in Leibniz's argument an anticipation of Wittgenstein's remark that Russell's paradox (as well as other confusions regarding the infinite) stem from a problematic extension of the notion of number from the context of finite numbers to that of infinite numbers.[17]

Leibniz's argument against the number of all numbers does seem to employ, though not explicitly, the self-referential aspect of this notion. As is well known, self-reference is endemic to some well-known paradoxes. We have seen that, in Russell's formulation, "the class of all classes which is not a member of itself", self-reference is essential to the paradox. Self-reference is most explicit in the liar paradox in its various formulations (e.g. "this sentence is false" or "what I am now saying is false"). It is also worth observing that Gödel incompleteness proof also employs self-referential claims. In fact, it is arguable that Gödel's proof is a very sophisticated mathematical recast of the liar paradox. In any event, it is clear that self-reference plays an important role not only in Russell's paradox but also in other important paradoxes and theorems.

Intuitively, it seems that the notion of "the number of all numbers" is self-referential. The syntactical form of the phrase strongly supports this intuition. Further, the semantics also speaks in favor of this intuition. After all, the subject of the phrase is a number, which numbers (or quantifies over) all numbers. Let us call this number N. Since (by hypothesis) N is a number, the phrase "the number of all numbers" must also apply to itself. In this sense, the phrase "the number of all numbers" is clearly self-referential. Likewise the phrase "the class of all classes" refers to itself in the sense that it includes itself. Thus Russell's formulation seems to make explicit what was implicit in Leibniz's formulation.

As I have already noted, Russell's antinomy of infinite number became a paradox in the context of the Frege-Russell programme. The goal of Russell's logicist programme was to define all concepts in logical terms. The definition of numbers was as difficult as it was crucial to the success of this programme. To define number in logical terms, Russell used classes as the logical objects and membership as the logical relation (Russell 1903: 166–7). As Lavine put it, "for Russell, a number was a class of all systems equinumerous to any member of the class. For example, on Russell's account, the number 2 is the class of all pairs" (Lavine 1994: 66).[18] When numbers were recast as classes, Russell's "antinomy of infinite number" became a severe problem for this programme. We have already noted that Russell articulated the problem in the form of the following contradiction: "there is and is not a greatest number". Once numbers are defined in terms of classes and the membership relation in a class, so that one can think of the number 2 as including the class of all pairs and the class of all units, it becomes natural to recast the notion of the number of all numbers as the class of all classes. Since the number of all numbers is to "number" all numbers, and, since it is itself a number, it also has to number or include itself or, in short, it has to refer to itself. In terms of classes, this notion is naturally rendered as a class of all classes that includes itself as a member. In this way, it is fairly easy to see how the self-reflexive character implicit in 'the number of all numbers' becomes explicit in Russell and Frege's attempt to define all numbers in terms of classes and membership in a class. By rendering Russell's formulation "there is and is not a number of all numbers" in terms of classes we would get the more straightforward paradoxical formulation: "there is and there is no class of all classes that do not belong to themselves."

Thus it seems that the definition of numbers in logical terms can naturally lead from Leibniz's argument to Russell's paradox. This also explains why it was such a devastating effect on the logicist programme of Russell and Frege. As we know, Frege was indeed entirely devastated – both personally and professionally. By contrast, Russell didn't give up the programme; rather, he set out to solve the paradox and save the programme. The result was his theory of types. Whether the theory of types saved the reductive programme is doubtful. In any event,

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the theory of types pays a heavy price for elegance and simplicity. In the end, it seems fair to conclude with Lavine that, "Russell's logicist programme failed as a result of the paradox" (Lavine 1994: 73).

3. THE GREATEST NUMBER AND THE GREATEST BEING

Thus far I have stressed the striking similarity between Leibniz and Russell's formulations. I would now like to show that they arose in very different contexts and play very different roles in each thinker's agenda. Let us turn to Leibniz's agenda. Leibniz's point that the number of all numbers is a contradiction makes its appearance in the context of an altogether different programme. In 1675–76, the period during which Leibniz develops his views about infinity (see Levey 1998), Leibniz is also engaged, among many other projects, in distinguishing possible and impossible notions. More precisely, in the texts from this period, Leibniz is attempting to demonstrate that some notions are possible while others are impossible. Leibniz presupposes a fairly crystallized theory of possibility. In brief, he identifies the possible with the thinkable or the conceivable in God's mind and he explicates the thinkable (or the intelligible) in terms of self-consistency among the terms of complex notions. Leibniz also presupposes a universally applicable method to distinguish possible notions from impossible ones. Leibniz's method can be stated roughly as follows: if the terms which compose a given notion are consistent *inter se*, then the notion indicates a possible thing; if the terms are inconsistent so that they imply a contradiction, the notion indicates an impossible thing. The method involves the analysis of complex notions into their constituents and in this way the determination of whether they involve internal contradictions.

It is very clear that Leibniz is using the notion of the number of all numbers in this context as an illustration of an impossible notion, i.e., one whose internal constituents imply a contradiction. For example, he states that the number of all numbers is a contradiction and goes on to discuss the twofold origin of impossibility. He writes: "The number of all numbers is a contradiction, i.e., there is no idea of it; for otherwise it would follow that the whole is equal to the part, or that there are as many numbers as there are square numbers" (A VI iii 463/ P 7). Immediately after that he writes that, "Impossible is a two-fold concept: that which does not have essence and that which does not have existence…" (A VI iii 463/ P 7). 'The number of all numbers' is an example of the first type of impossibility.[19]

At the same time, he is also using the notion of the number of all numbers in a more specific context, namely *in contrast* to a notion whose possibility he is very keen to prove, namely, the notion of the greatest or the most perfect being (*Ens Pefectissimum*) (A VI iii, 572/ P 91).[20] His objective in this context is to support Anselm's argument, revived by Descartes, according to which God exists since existence is included in his notion as one of his perfections. For Anselm's argument to be valid, one has to show that the definition of the greatest being is possible. As he writes, "God is a being from whose possibility (or, from whose essence) his existence follows. If a God defined in this way is possible, it follows that he exists"

(A VI iii, 582/ P 105). Descartes, as well as the rest of the tradition, simply assumed that the definition of the most perfect being is non-problematic. Leibniz points out that this supposition requires proof. In *A Specimen of Discoveries* (c.1686), Leibniz writes:

> A real definition is one according to which it is established that the defined thing is possible, and does not imply a contradiction. For if this is not established for a given thing, then no reasoning can be safely taken about it, since if it involves a contradiction, the opposite can perhaps be concluded about the same thing with equal right. And this was the defect in Anselm's demonstration, revived by Descartes, that the most perfect or the greatest being must exist, since it involves existence. For it is assumed without proof that a most perfect being does not imply a contradiction; and this gave me occasion to recognize what the nature of real definition was. (RA 305–7)

While Leibniz's preoccupation with the notion of the greatest being is familiar, it has not been recognized that Leibniz traded on the connection between the possibility of the greatest being and the impossibility of the greatest number. I will try to show that this connection is evident – both textually and conceptually – and that it has very interesting implications. Leibniz defines the notion of the most perfect being as "the subject of all perfections" (A VI iii, 580/ P 103) – "one which contains all essence, or which has all qualities, or all affirmative attributes" and attempts to demonstrate that this notion "is possible or (*seu*) does not imply a contradiction" (A VI iii, 572/ P 91). It appears in these formulations that the notion of God as the greatest being is closely related to the notion of a totality taken in a quantitative sense; for it is defined as that which contains all essence, all perfections, all qualities or all affirmative attributes. In short, the notion of God is defined in quantitative terms and as the subject of *all* perfections or attributes.

Now we might wonder why Leibniz is anxious to show that such a notion is possible. Why does he see it as something requiring a proof? Why should the possibility of the greatest being – a traditionally accepted and apparently innocuous notion – be in question at all? After all, Leibniz fully accepts the traditional view of God as entailing all knowledge, as entailing all power, all wisdom and all Being.

A general answer to this question derives from what I have already mentioned, namely that Leibniz was deeply committed to the project of distinguishing between possible and impossible notions by analyzing complex concepts into their constituents and examining their internal consistency. This general reply, however, does not explain Leibniz's particular interest in proving the possibility of the notion of the greatest being, which otherwise would seem non-problematic. This is why a more specific reply is needed. I suggest the following: Leibniz is concerned about the possibility of the notion of a totality and, in particular, about God as the maximal totality because he clearly sees that similar notions, namely, the notion of the greatest number (and that of the most rapid motion and the greatest shape) are problematic. The syntactical similarity between the notion of the greatest number, seen as the totality of all numbers, and that of the greatest being, seen as the totality of all perfections, is clear. Hence it is likely to have evoked Leibniz's intellectual concerns about the traditional notion of God.[21] It hardly needs mentioning that, if it turned out the notion of the *Ens Perfectissimum* would be inconsistent, disastrous consequences would follow: not only for rational theology in general but also for the very foundations of Leibniz's own metaphysics.[22]

That the relations between the notions of the greatest being and the greatest number concern Leibniz is evident. He juxtaposes and contrasts these definitions in the very same papers and notes from the Paris writings (e.g. A VI iii, 520/ P 79). In this very paper Leibniz explicitly draws an analogy between the essence of God and the essence of the number 6 as being composed of six units (A VI iii, 518/ P 77). He is clearly toying with the analogy between God as the subject of all perfections and number as subject of units. Direct evidence for Leibniz connecting the notions of the greatest being and the greatest number appears in a later text. He wrote explicitly that Descartes agrees to the analogy between these notions:

> Mons. Des Cartes in his reply to the second objections, article two, agrees to the analogy between the most perfect Being and the greatest number, denying that this number implies a contradiction.[23] It is, however, easy to prove it. For the greatest number is the same as the number of all units. But the number of all units is the same as the number of all numbers (for any unit added to the previous ones always makes a new number). But the number of all numbers implies a contradiction, which I show thus: To any number, there is a corresponding number equal to its double. Therefore, the number of all numbers is not greater than the number of all evens, i.e., the whole is not greater than its part. (GP I, 338; cited from Russell's appendix, 244)

It seems reasonable to suppose that Leibniz's clarity about the impossibility of the greatest number (as well the most rapid motion and the greatest shape) plays a role in his concerns about the possibility of the greatest being – which is partly why its possibility required a proof in the first place.

In any event, it is clear that Leibniz is investigating these notions by comparing and contrasting them. In Leibniz's eyes, these examples provide paradigmatic cases of possible versus impossible notions. It is also clear that each of these notions is of great consequence to Leibniz's philosophy. For this reason, the relations between them are all the more interesting. Yet it is very curios that, while these concepts have a striking structural similarity and both seem to imply infinite quantity the concept of the greatest being serves Leibniz as a paradigm of a *possible* notion and the notion of the greatest number serves as a paradigm of an *impossible* notion.

Since these notions seem analogous, Leibniz's position is very intriguing. What makes the notion of the greatest being a paradigm of possibility and that of the greatest number a paradigm of impossibility? As we shall see, the distinction between these notions points to a deep insight in Leibniz's metaphysics. To see this, let us try to advance the analogy a bit further. As we noted, Leibniz analyzed the notion of the greatest being in quantitative terms, i.e. as "the subject of all perfections" (A VI iii, 580/ P 103), "one which contains all essence, or which has all qualities, or all affirmative attributes." In the same texts he also draws an explicit analogy between God's essence and whole numbers.[24] In this analogy, numbers consist of units as God's essence consists of simple forms or perfections. Since Leibniz defines whole number as consisting of units, the greatest number is seen as including all units. Since he defines God as consisting of all essence or all perfections, the greatest being is seen as the subject of all perfections. Just as there are infinitely many units in the notion of infinite number, so there are infinitely many perfections in the notion of God. In this sense, these notions seem perfectly analogous. Therefore, it seems that they should be considered to be equally problematic. Yet we have seen that Leibniz considers the one as a paradigm of possibility, the other as a paradigm of impossibility. What then is the dissimilarity Leibniz sees between these notions? What makes him consider the one notion to be possible and the other to be impossible?

Let me make a conjecture. In spite of the close similarity between these notions, there is in fact substantial difference between them. The dissimilarity stems from the difference between beings and numbers – a distinction that cuts deep in Leibniz's metaphysics. We know that Leibniz does not consider numbers to be true beings. As he writes, "Numbers, modes, and relations are not entities" (A VI iii, 463/ P 7). A major difference between these notions might hinge on the distinction between the concept of the greatest being and that of a greatest non-being. While numbers are universal, non-active, and not true units, beings for Leibniz, are individual, active units. In short, beings, for Leibniz, are agents.

Furthermore, the notion of God or the greatest being serves as the paradigm of Being. It is the first being and the source of all created beings. In fact, it also serves as the model for Leibniz's notion of created beings – individuals that have power and internal source of activity. Even so, the question why the notion of the greatest being, seen as consisting of infinitely many perfections, is possible stands. Let us not forget that Leibniz's strategy to prove that the greatest being is possible is to show that all positive perfections or attributes are compatible *inter se* and therefore may be included in one subject. So, how is such a notion possible if the notion of infinite number is not?

In fact, this is precisely what becomes clear when we compare the notion of God to that of infinite number. Unlike the notion of a number, the notion of God (and, if fact, of any true being) is not additive; it is not *composed* of infinite units or of perfections. It is not a sum of all perfections; rather, it is initially a *subject* which includes all perfections. In this context, the notion of a subject seems to indicate individuality, unity and activity.[25] Unlike numbers, ideas and other incomplete notions, subjects act. Subjects, for Leibniz, are agents. God, of course, is the primary agent. This indicates that, unlike the notion of number, the notion of God is not purely quantitative. The source of being, according to Leibniz, is intrinsic activity. God's intrinsic activity is also the source of its unity and perfection. In fact, the notion of the *Ens Perfectissimum* is more accurately rendered as the highest being or the most perfect being, which points that the highest or most perfect being need not pertain primarily to a quantitative aspect but rather to a qualitative one.

This also clarifies the grounds for Leibniz distinction between true entities and aggregates. Beings or true unities are not composed. He writes, for example, that, "…no entity that is truly one [*ens vere unum*] is composed of parts. Every substance is indivisible and whatever has parts is not an entity but only a phenomenon".[26] This distinction becomes all the more significant when we consider the context of infinity. The context of infinity clarifies that the greatest number is impossible while and greatest being is possible. Unlike a number, a being is not defined quantitatively or compositionally; rather, it is defined through its basic ability to act.[27] Similarly, Leibniz defines created beings (as well as their infinite concepts) by their unique method of production or law of formation, not as a sum of their predicates.[28] This is why Leibniz can accept infinite beings while rejecting infinite numbers. In this light, it becomes rather clear why Leibniz states the following: "It is not surprising that the number of all numbers (*numerum omnium numerorum*), all possibilities, all relations or reflections, are not distinctly understood; for they are imaginary and have nothing that corresponds to them in reality" (A 399/ P 115). AQ3

NOTES

[1] Consider his book *Introduction to Mathematical Philosophy* (1919) and his explicit desire to establish a school of Mathematical Philosophy as evidence.

[2] Leibniz wrote: "Ma Metaphysique est toute mathematique, pour dire anisi, ou la pourroit devenir" (Letter to de l'Hospital, GM II, 258, cited from Couturat 1901: 281–2.

[3] I thank Meir Buzaglo for an illuminating discussion of this point.

[4] In his *A Critical Exposition of the Philosophy of Leibniz*, Russell described Leibniz as attempting to reduce all relations and polyadic predicates to monadic ones. Since such a reduction has shown to be formally impossible, Russell argued that Leibniz's system fails.

[5] From a perspective of the history of philosophy in the English-speaking world (which has of course enormous influence on the philosophical world at large) this point is evidenced by two simple observations: (1) Logic – that is, mathematical Logic – has become an obligatory course in almost any academic philosophical training (so that formal logic became an essential part of a philosopher's knowledge and one of his or her basic working tools). (2) The logic taught in the basic philosophy courses is the logic of relations introduced by Russell and Frege.

[6] Letter to Russell, cited from Lavine 1994: 55. See p. 293 for a translation of the whole letter.

[7] This chronology is mainly based on two sources: G. H. Moore's introduction to RCP and Lavine 1994.

[8] Lavine 1994: 56–7.

[9] In his *Autobiography* (Russell 1967–69: 200) he later wrote: "At the time I falsely supposed all his arguments to be fallacious, but I nevertheless went through them in the minutest detail. This stood me in a good stead when later on I discovered that all the fallacies were mine." (See Lavine 1994: 57).

[10] Incidentally, the majority of his correspondence with Moore at the time deals with questions of these translations. Russell also corresponds with Louis Couturat who invites him to participate in the Paris conference, where his first encounter with Peano (mentioned above) takes place.

[11] Moore goes on to remark: "Here again, he [Russell] stood at the brink of the Paradox of the Largest Cardinal" (RCP xxiii).

[12] Here is another one of Russell's formulations of this point: "…there is a number after any given number and therefore no number N that may be specified is the number of all numbers" (RCP III, 32).

[13] See Moore and Garciadiego 1981.

[14] For which he cites GP VI, 629; GP I, 338; GP II, 304–5; GP V, 144; Langley ed. 1896: 161 (Russell, 1937: 111).

[15] See also the following argument from 1672: "Or perhaps we should say, distinguishing among infinites, that the most infinite, or all the numbers, is something that implies a contradiction, for it were a whole it could be understood as made up of all the numbers continuing to infinity, and would be much greater than all other numbers that is, greater than the greatest number" (A VI iii, 168/ RA 116).

[16] This point is due to Haim Gaifman.

[17] See Shanker 1987: 187. Wittgenstein wrote: "It seems to me that we can't use generality – all, etc. – in mathematics at all. There's no such thing as 'all numbers', simply because there are infinitely many". Wittgenstein 1974: §126.

[18] Russell and Frege adopt the extensional interpretation of numbers because they attempt to reduce all concepts (particularly, number concepts) to logical concepts. But in response to the question "What is numbered?" they cannot refer to any non-logical objects. For this reason they use equi-numerousity between classes as their basic notion. On this see Lavine 1994: 65–6. Lavine writes: Russell was a logicist. He wished to show that mathematics and logic are one by showing how to develop all of mathematics within a framework free of any special conditions or empirical and psychological assumptions. That is a programme substantially similar to Frege's for arithmetic and analysis. Frege and Russell faced a common problem: mathematics is apparently about objects (numbers and so forth), and yet the assumption that objects exist apparently goes beyond logic […] Russell used classes as the logical objects and membership as the logical relation (Russell 1903: 166–7)".

[19] I do not discuss in this paper the other (very interesting) type of impossibility.

[20] In a letter to Conring (1677) Leibniz writes: "*At qui subtiliores sunt adversarii ajunt Ens perfectissumum tam implicare contraditionem quam numerum maximum*" (A II i, 325).

[21] "There cannot be a most rapid motion or a greatest number. For number is something discrete, where the whole is not prior to its parts, but conversely. There cannot be a most rapid motion, because motion is a modification, and is the transference of a certain thing in a certain time. (Just as there cannot be a greatest shape.) There cannot be one motion of the whole, *but there can be a kind of thinking of all things*. Whenever the whole is prior to its parts, then it is a maximum, as in space and in a continuum. If matter is like a shape, namely that which makes a modification, then it seems that there is no totality of matter" (A VI iii, 520/ P 79, my italics).

[22] In 1678, Leibniz writes to Elizabeth: "Mais à présent, il me suffit de remarquer, que ce qui est le fondement de ma caractéristique l'est aussi de la demonstration de l'existence de Dieu" (A II i, 437).

[23] In his second objection to Descartes's *Meditations* Caterus argued that humans may invent or think out the concept of the greatest being from their own resources, just as they may think the concept of the greatest number though it is impossible.

[24] "It seems to me that the origin of things from God is of the same kind as the origin of properties from an essence; just as $6 = 1+1+1+1+1+1$, therefore $6 = 3+3, = 3\times 2, = 4+2$, etc. […] So just as these properties differ from each other and from essence, so do things differ from each other and from God" (A VI iii, 518–9/ P 77. See also A VI iii, 523/ P 83; A VI iii, 512/ P 67 for similar analogies and A VI iii, 521/ P 81).

[25] See Fichant 1997.

[26] Cited from Brown, G. 2000: 41.

[27] This point might have interesting bearing on the debate between Richard Arthur and Gregory Brown (in *The Leibniz Review* 1998, 1999, 2000, 2001) regarding Leibniz's denial of infinite number and infinite whole. The question I discuss above, what justifies Leibniz to regard the notion of an infinite being as possible and that of an infinite number impossible is at the background of the debate between Arthur and Brown.

[28] A very interesting corollary to this view is Leibniz's definition of infinite series. He does not define infinite series as a sum of numbers but as a product of its formation rule. In this connection see the interesting discussion in Couturat 1973: 476. Couturat cites this passage from the letter to des Bosses (of 11 March 1706): "*Neque enim negari potest, omnium numerorum possibilium naturas revera dari, saltem in divina mente, adeoque numerorum multidudinem esse infinitiam*" (cited from Couturat, 1973: 476).